Source estimation for emergency response

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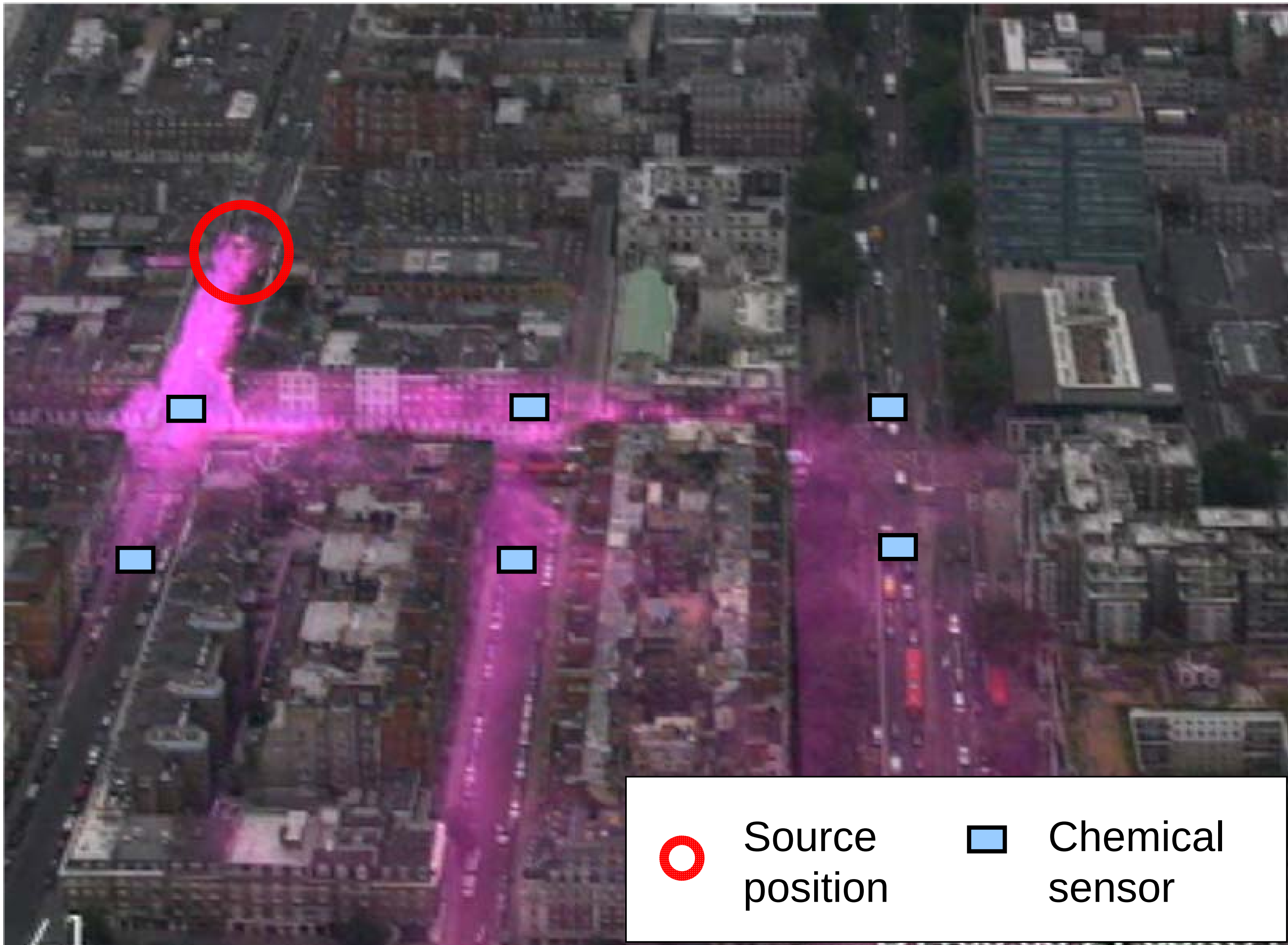
The problem…



The DYCE consortium

Broadband chemical sensors

OWLSTONE

Communications & networking

SELEX GALILEO

Inverse modelling to estimate the source characteristics

Wind tunnel validation experiments

Uncertainty analysis

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Funding EPSRC

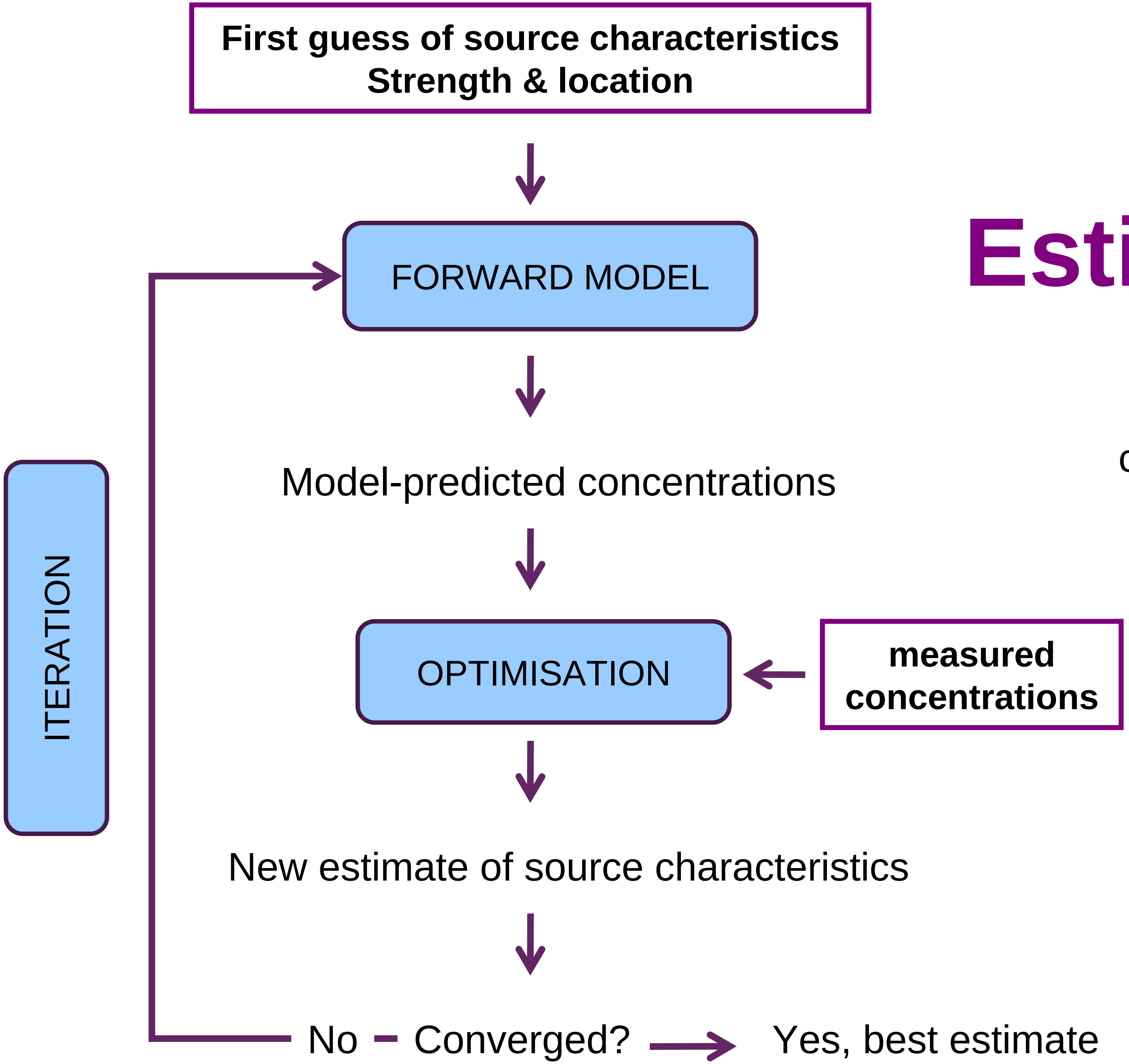
Engineering and Physical Sciences Research Council

Technology Strategy Board

Driving Innovation

Source Estimation

Inverse problem: estimate source characteristics from concentration measurements

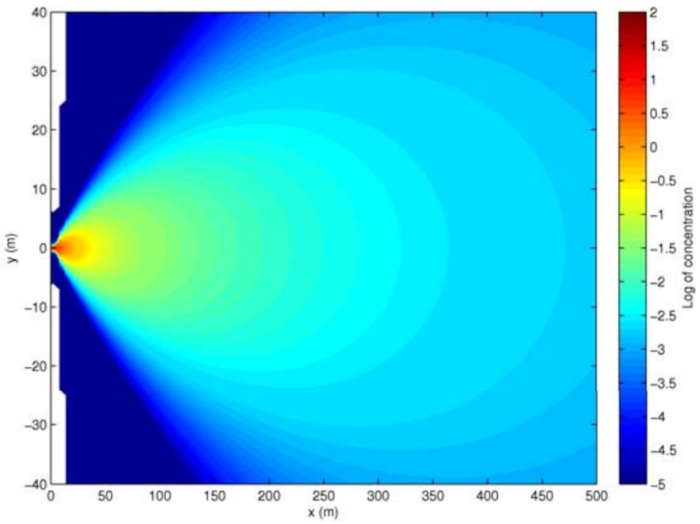


Forward model - Gaussian plume model

Model used to generate the model-predicted concentrations

Input

- source strength Q
- source location (X_s, Y_s, Z_s)
- wind speed u
- stability
- sensor locations

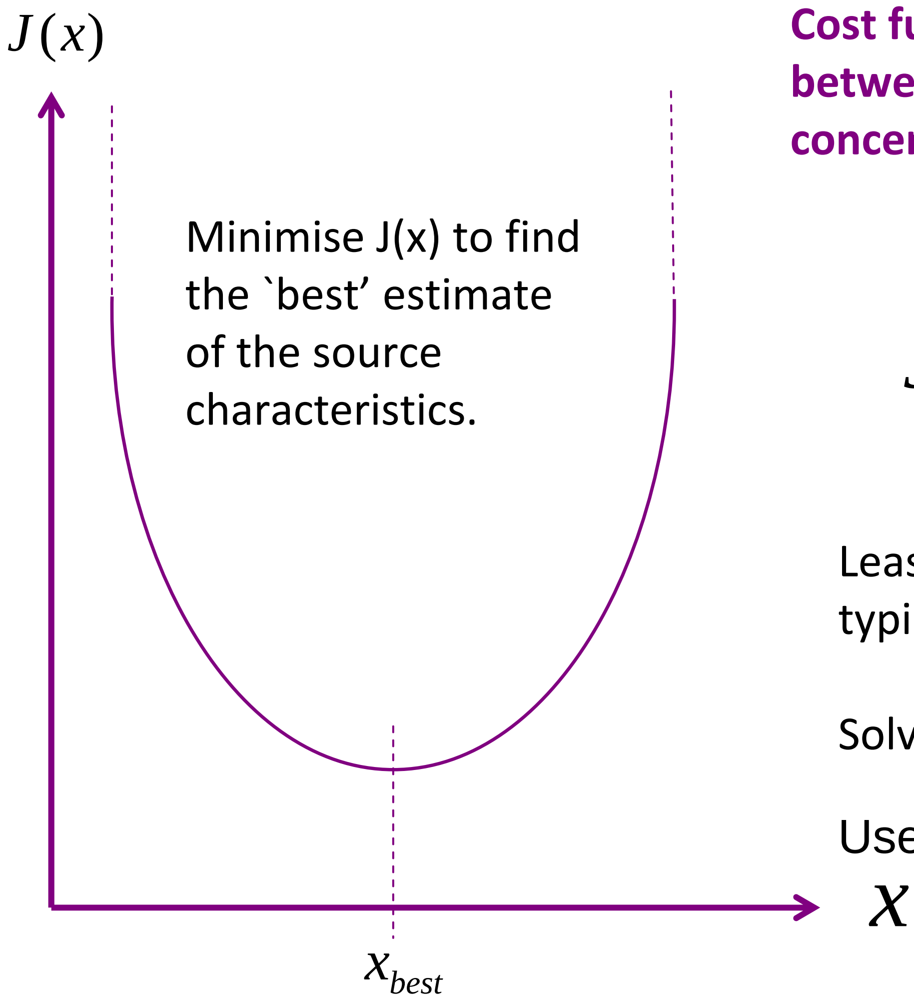


Assumptions;

- Continuous emission from ground-level source at constant rate
- Steady state flow and constant meteorological conditions
- Neutral conditions
- Dispersion over level, open terrain

Optimisation

An optimal estimate obtained by minimising a cost function starting from a first guess



Cost function, *J(x)*: measures the discrepancy between the measured and model-predicted concentrations

$$J(x) = \frac{1}{2}\sum_{i=1}^{N}\frac{\left(C_i^o - C_i^m\right)^2}{\sigma_i^2}$$

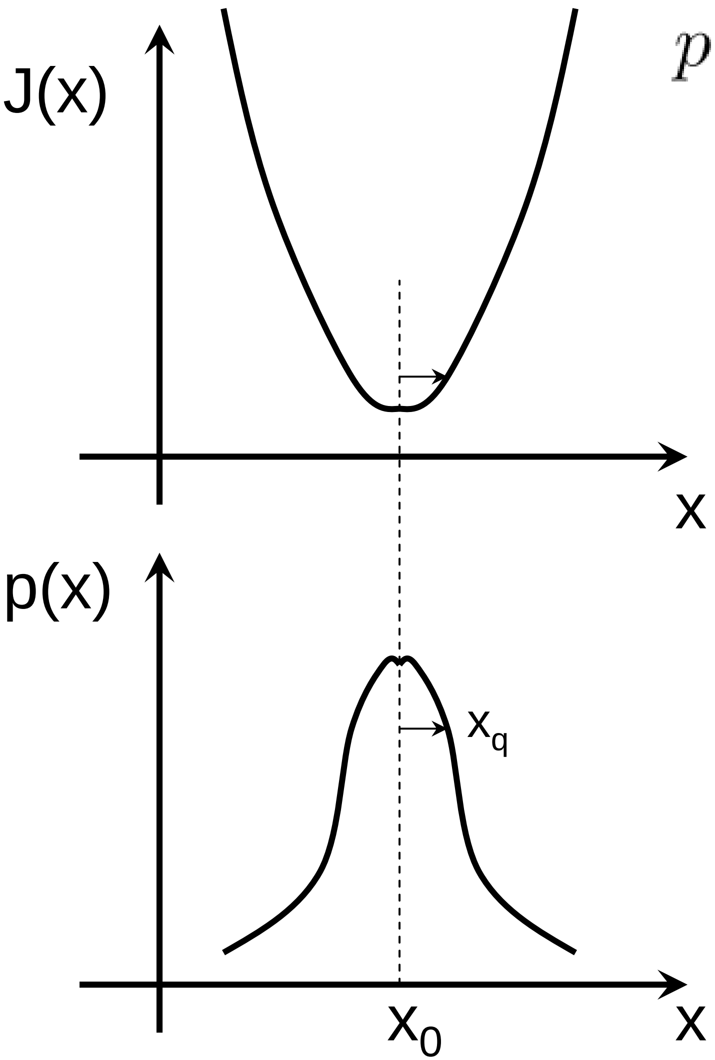
Least squares fit plus error weighting, typically observational and model error

Solve for $J'(\mathbf{x}) = 0$

Use Gauss-Newton root finding method

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \frac{J'}{J''}$$

Estimating the quality of the analysis



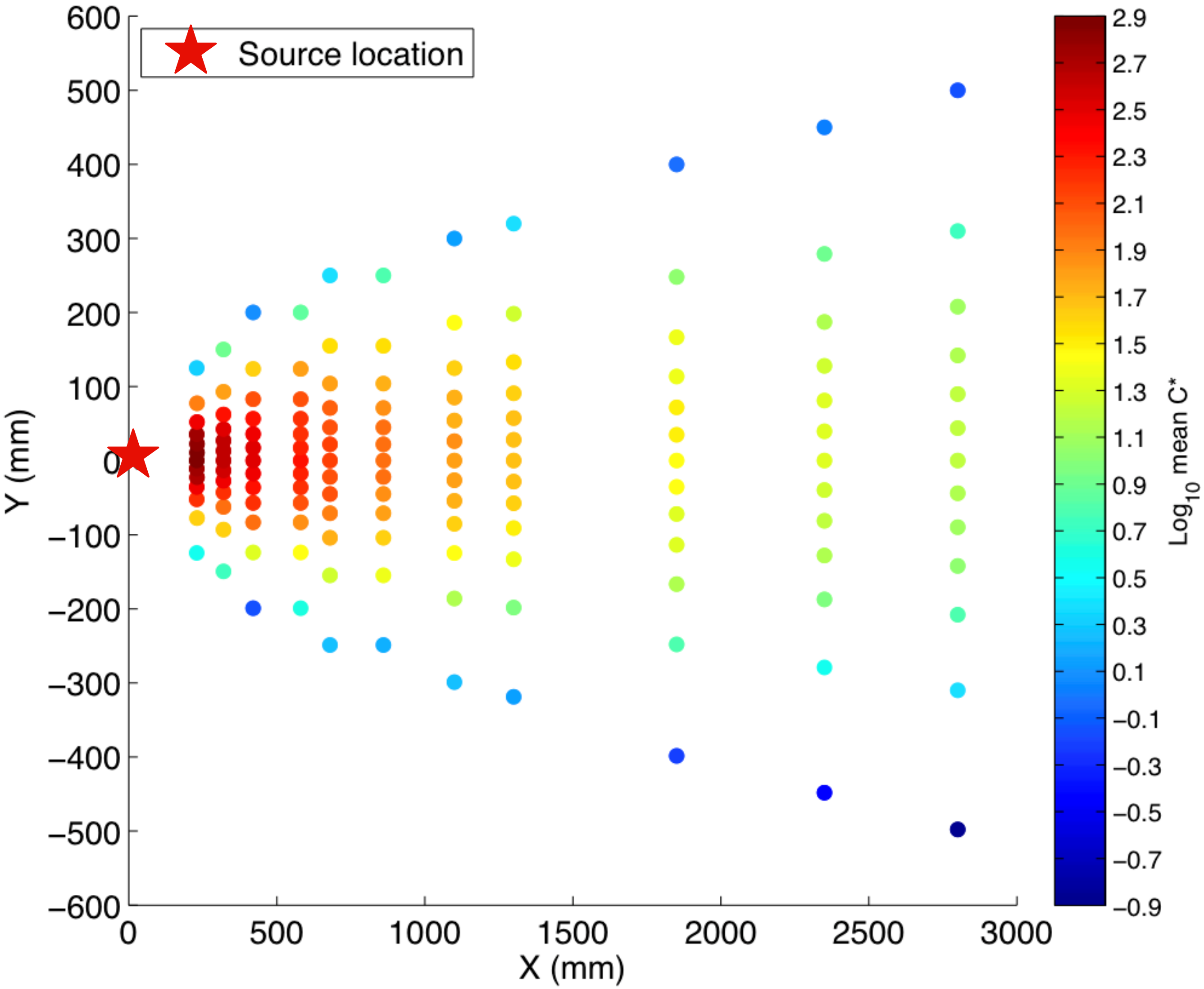
$$p(x) = \exp(-J(x))$$
$$\approx \exp\{-J(x_0) - \frac{1}{2}x^2 J''(x_0)\}$$

Error, or quality estimate:

$$x_q = (2/J''(x_0))^{\frac{1}{2}}$$

For vector of states, **x**, use diagonal elements of *J''(x₀)*

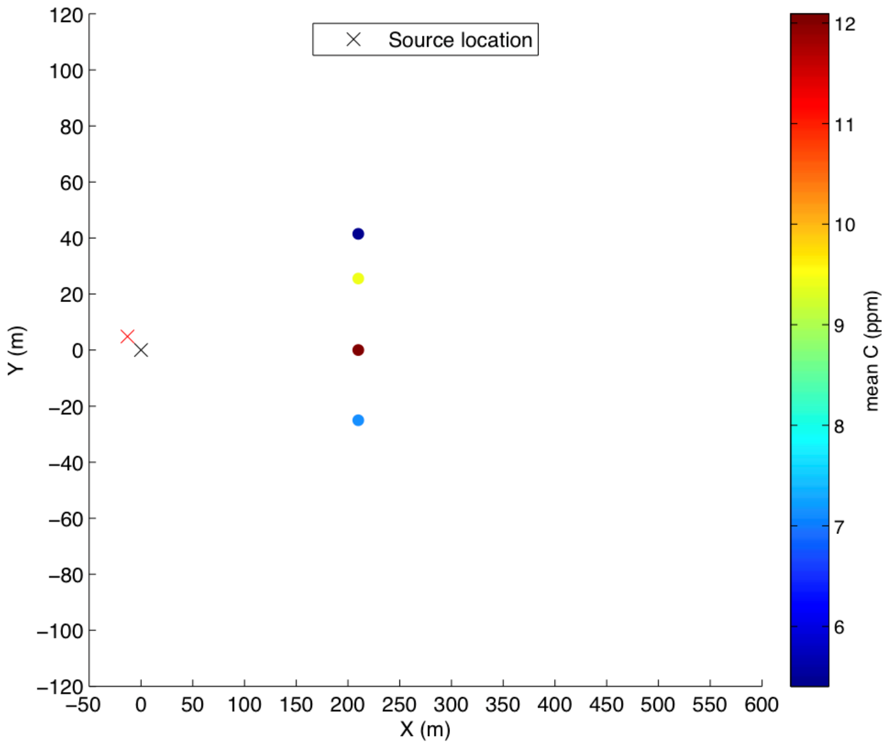
Wind tunnel measurements



Inverse modelling example

Source parameter	True value	First guess	units
Q	0.1	0.7	$m^3\ s^{-1}$
Xs	0	10	m
Ys	0	10	m

Source parameter	Estimate	Uncertainty	units
Q	0.11	0.01	$m^3\ s^{-1}$
Xs	-13.0	13	m
Ys	4.8	1.0	m



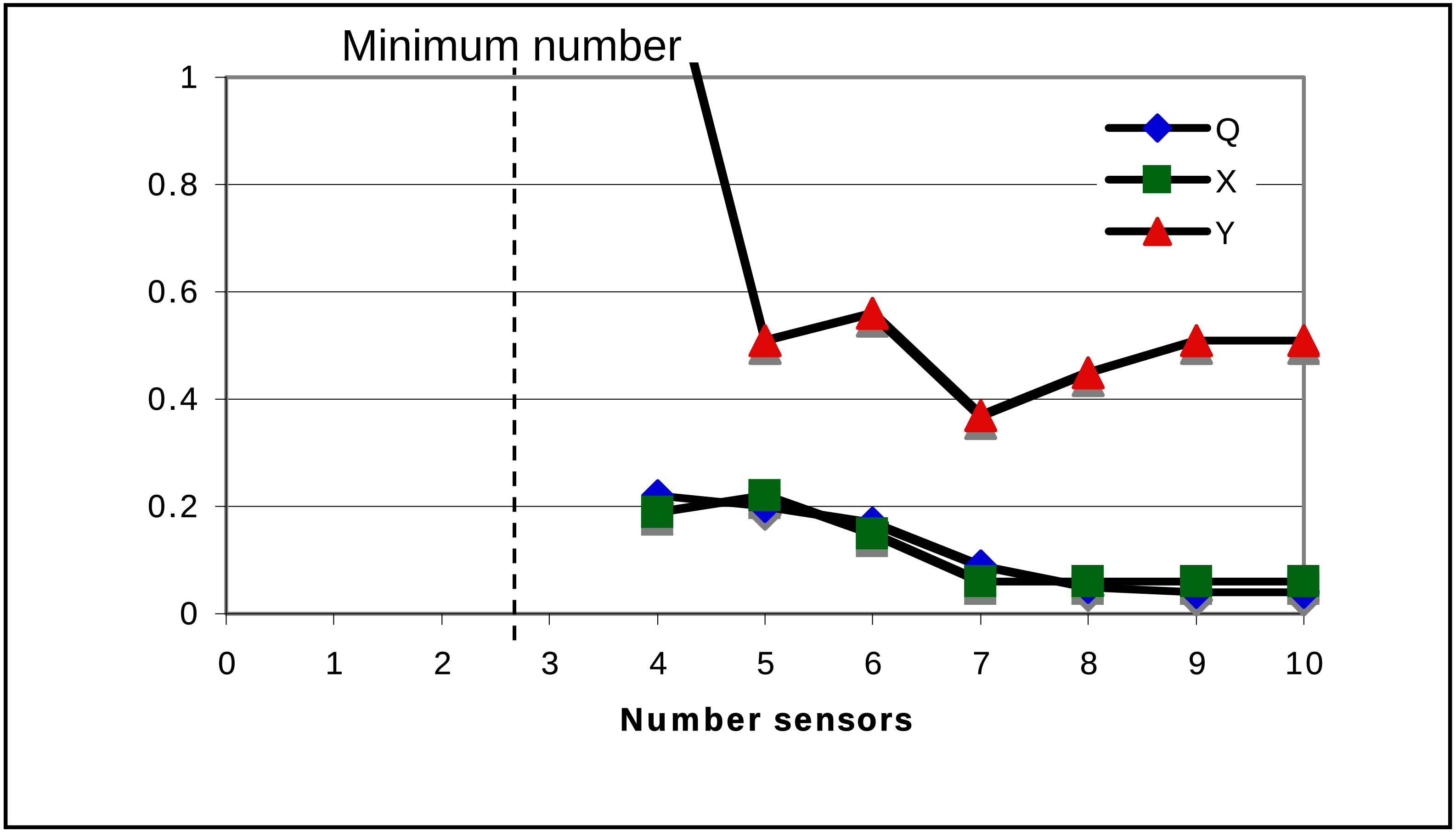
4 sensors:

Mean X distance between source and sensors = 210 m

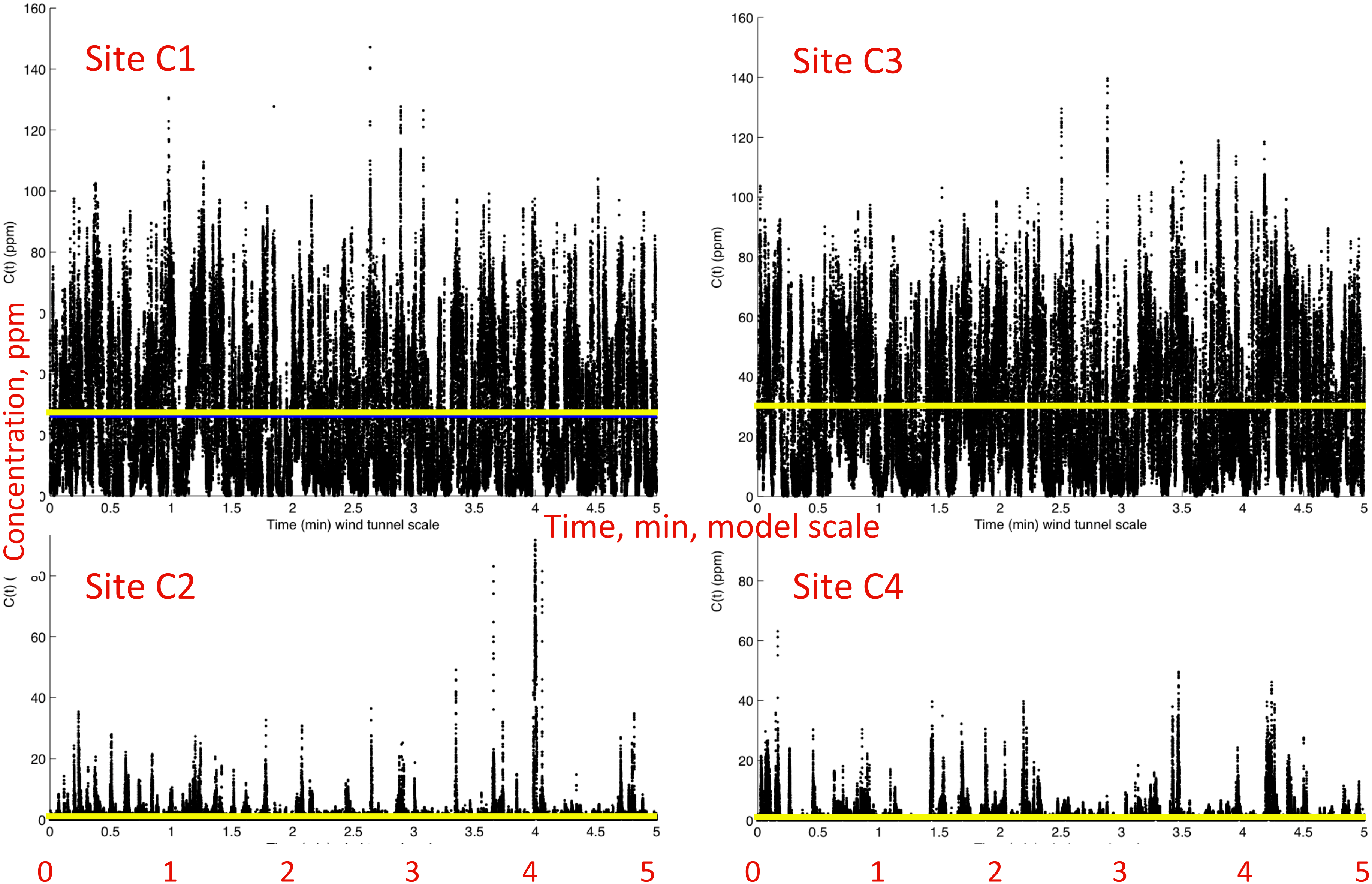
Mean Y distance between source and sensors = 31 m

Number of sensors

Errors relative to mean distance of sensors to source



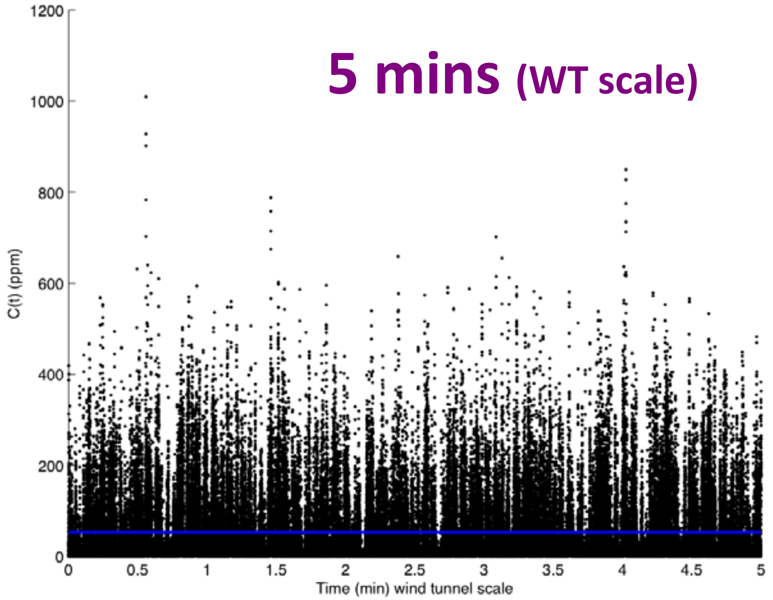
4FFID - example time series data



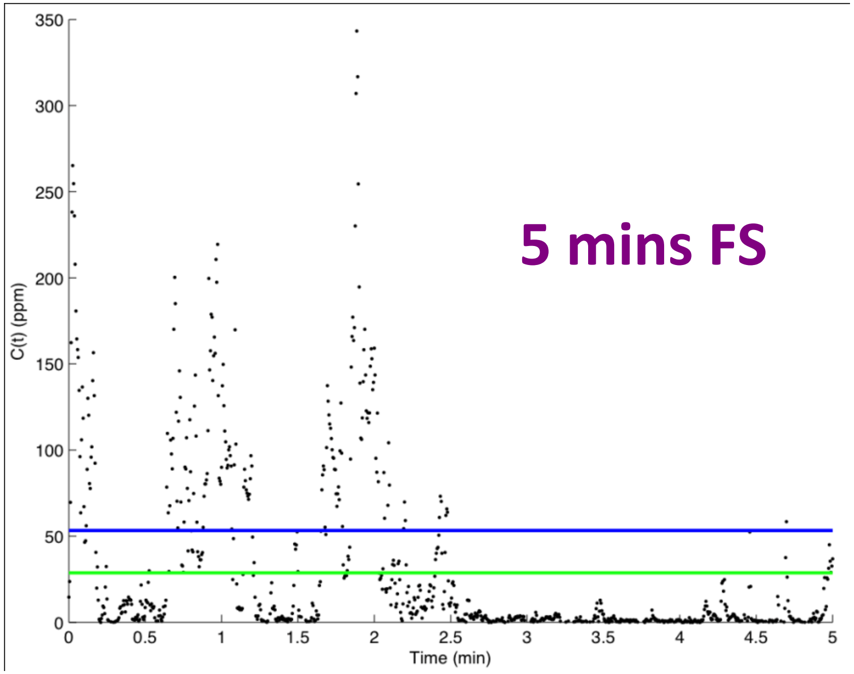
Sources of error/uncertainty

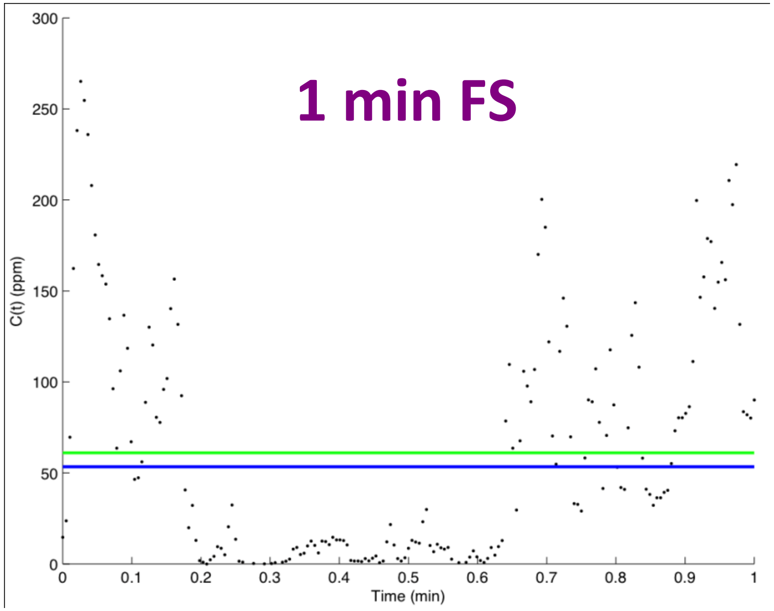
- **Measurement error** the accuracy of the concentration measurement from the sensor ***may be known***
- **Model error** how good is the model at representing reality? ***can only estimate***
- **Sampling error** this is dependent on the averaging time of the data due to the natural variability of the concentrations ***likely to dominate***

Shorter time traces



`true' mean
shorter averaged time mean





5 mins at WT scale is about 10 hours at full scale!

Sampling error

Quantify sampling error:

Short time average to estimate the true mean in a turbulent flow

Standard deviation of the shorter time mean estimate of the true mean concentration

$$\sigma_{\overline{C}^t} = \left(\frac{1}{n} \sum_{i=1}^{n} \left(\overline{C}_i^t - \overline{C}^T \right)^2 \right)^{\frac{1}{2}}$$

t is the shorter averaging time

T is the total time length

n is the number of shorter averaging time samples

$\overline{C}_i^t$ = mean concentration averaged over time t

$\overline{C}^T$ = true mean concentration

Sampling error

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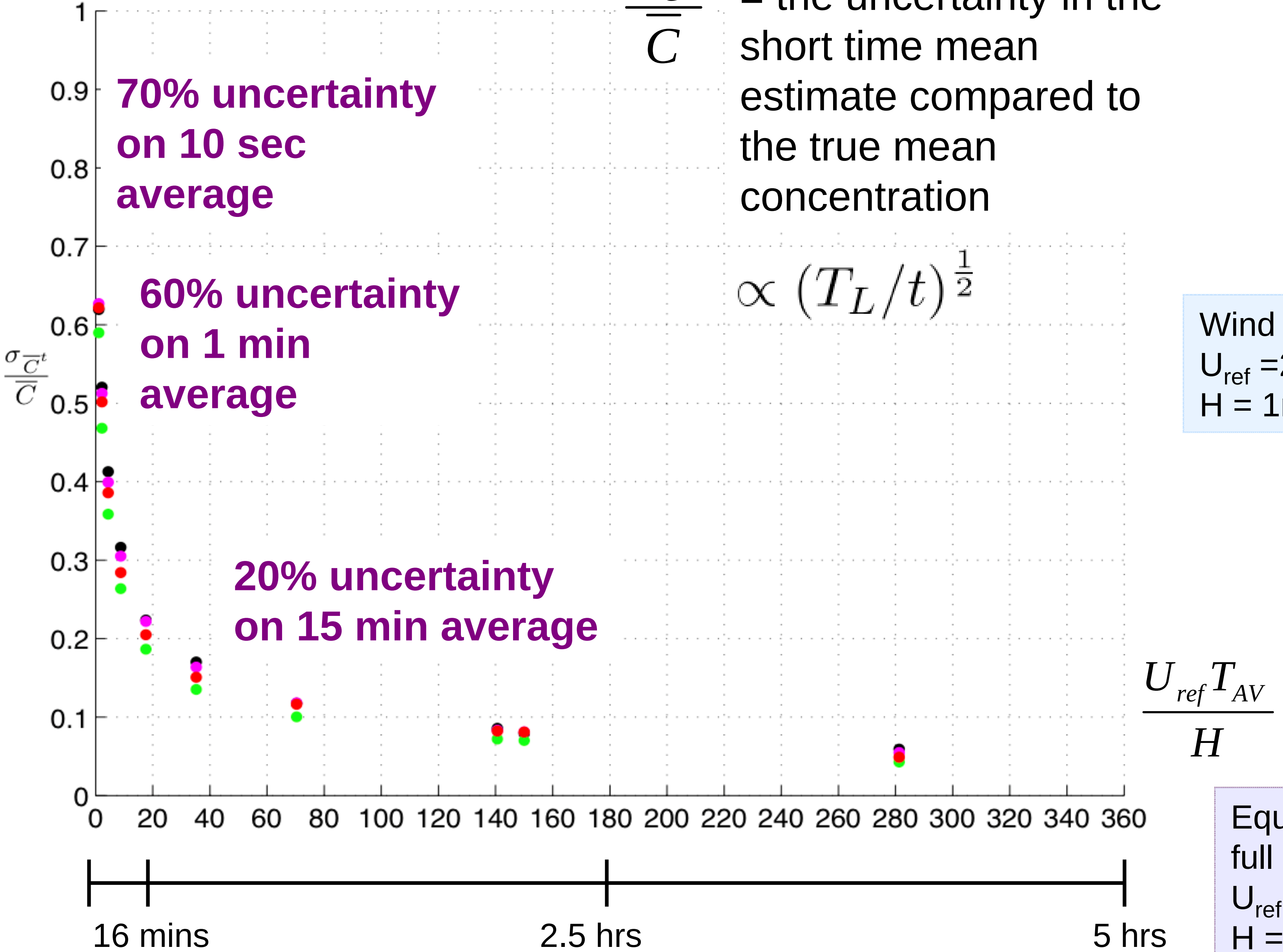
$$\frac{\sigma_{\overline{C}^t}}{\overline{C}}$$ = the uncertainty in the short time mean estimate compared to the true mean concentration

$$\propto (T_L/t)^{\frac{1}{2}}$$

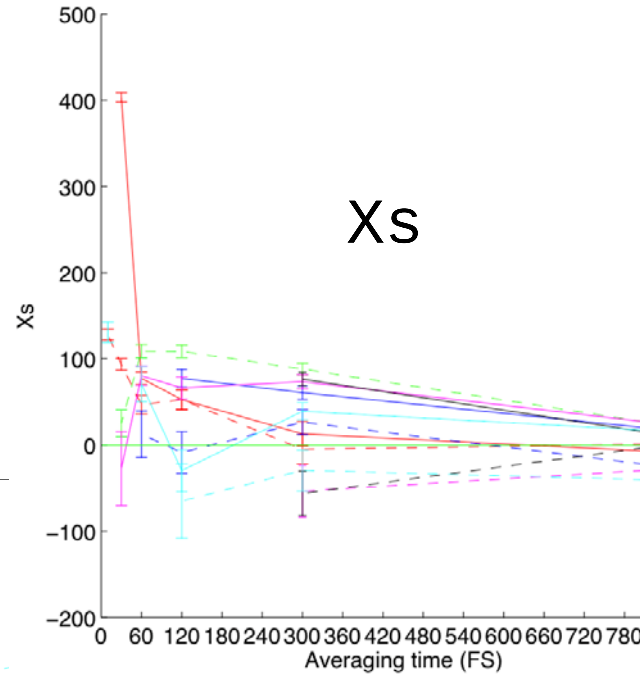


Wind tunnel
U_{ref} =2.5 m/s
H = 1m

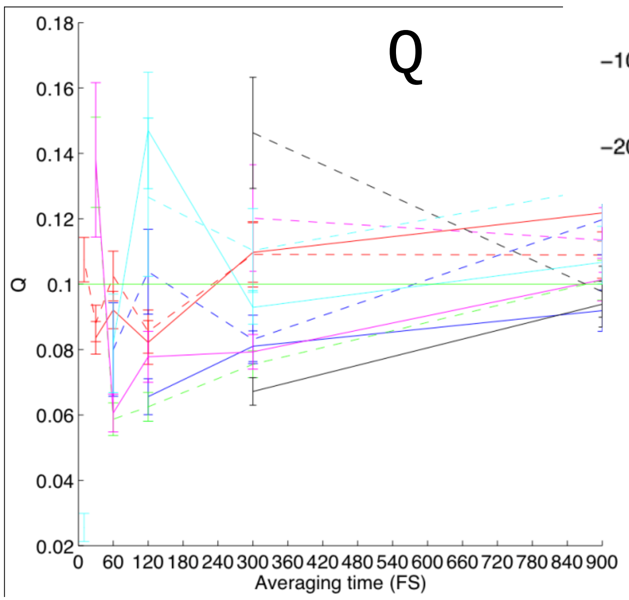
Equivalent full scale
U_{ref} =10 m/s
H = 500m

Ensemble

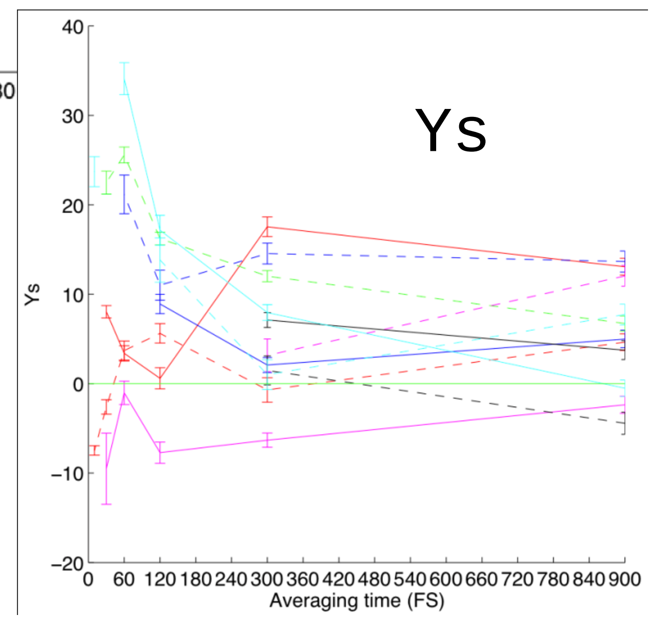
Ensemble of shorter averaged wind tunnel data and effect on the estimate of the source characteristics



Error bars represent the uncertainty of the best estimate



Each line is a different ensemble member



Model error also large at short times

Conclusions

- Developed and tested inverse algorithm
  - Need at least 3 sensors; Better with 5-7 sensors
  - performs best for configurations with sensors across the plume
  - ideally at least 3 sensors across the plume
- Sampling error is likely to be dominant
  - Pressure for quick answers; Perhaps do not need to be accurate?
  - Do need to be consistent with longer term averages
- Future
  - Need to quantify source errors when sampling error large
  - Develop method with network model
  - Use inverse framework for model parameter estimation
  - Intelligent algorithms for iterating sensors placement
  - Unsteady sources - sampling issues even more critical

Comparison with DNS

$$C_{i,j} = a\, C_{i-1,j} + b\, C_{i,j-1} + c\, C_{i-1,j-1} + Q_{i,j}$$

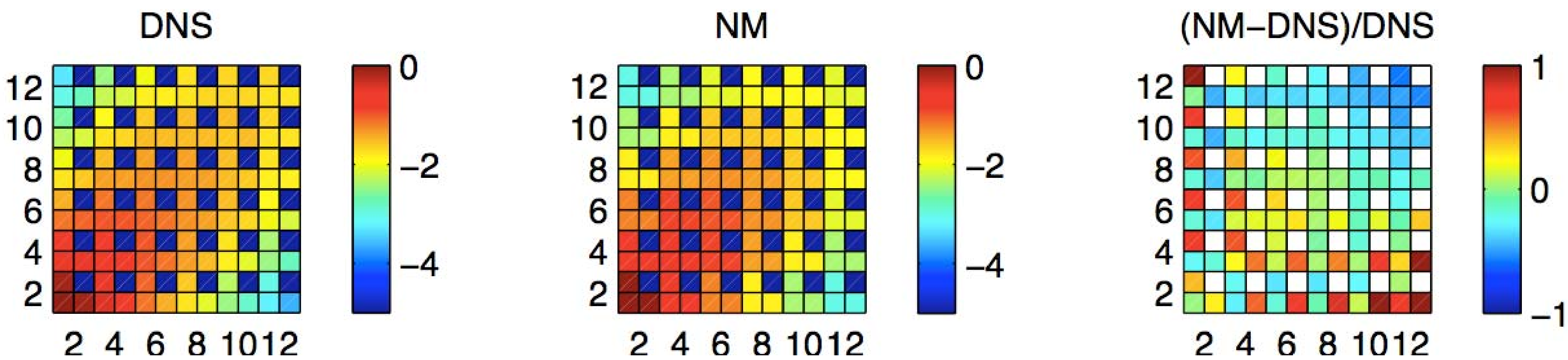
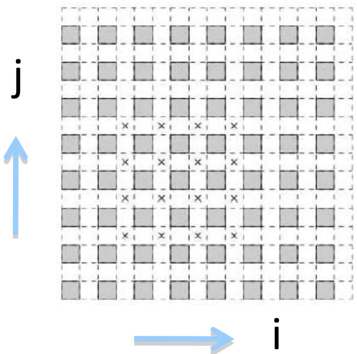
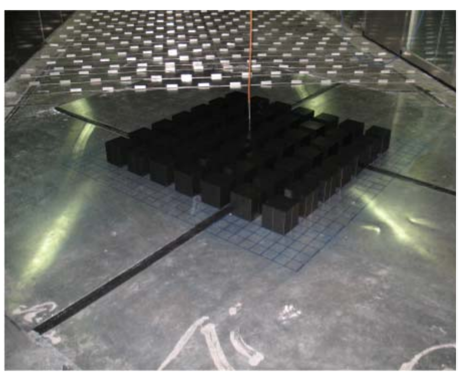


Figure 7: Comparison of concentration profile from a DNS with the network model solution. (a) $\log_{10} C_{i,j}^{DNS}$ for the DNS, (b) $\log_{10} C_{i,j}^{NM}$ for the network model, with $a = b = 0.25$, $r = s = 1$ and $c = 0.15$, normalised difference $(C_{i,j}^{NM} - C_{i,j}^{DNS})/C_{i,j}^{DNS}$. Concentrations are normalised by that in the first cell, where the source is located. The wind forcing is at 45°, along the leading diagonal.

Comparison with DNS and WT data



Branford et al, 2011



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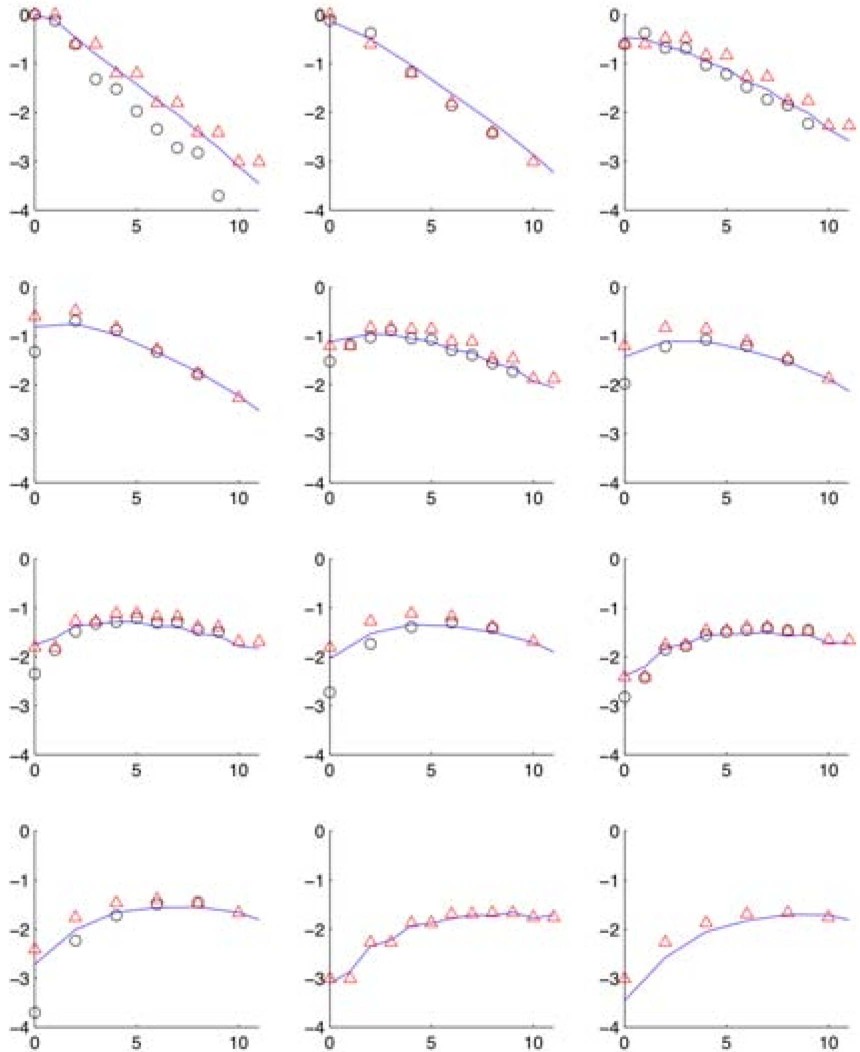

log(C/C0) vs. i for different j

Blue line: DNS

Black circles: WT data

Red triangles: NM

45 deg flow

a = b = 0.25

c = 0.2

C0 is concentration in source cell; i and j are node indices (i = j = 0 denotes the source cell)